ON WHY NON NORMAL MODALITIES. PLURALISM FOR A MONIST AND THE CASE OF THE COUNTERLOGICAL

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Abstract

The aim of the paper is to offer a dialogical interpretation of non-normal modal logic which will suggest some explorations beyond the concept of non-normality. This interpretation will be connected to the discussion of a minimalist defence of logical pluralism.

Keywords: non-normal modal logics; dialogical logic; logical pluralism.

Convincitur ergo etiam insipiens esse vel in intellectu …
Anselm of Canterbury,
Proslogion, capitulum II, Ps 13, 1, 52, 1

(Thus, even he who knows no better will be convinced that
at least it is in the intellect…)

At the end of the 19th century Hugh MacColl (1837-1909), the father of pluralism in formal logic, attempted in the north of France (Boulogne sur mer) to formulate a modal logic which would challenge the semantics of material implication of the post-Boolean wave. It seems that in some of his various attempts MacColl suggested some systems where the rule of necessitation fails.[i] Moreover, the idea that no logical necessity has universal scope - or that no logic could be applied to any argumentative context - seems to be akin and perhaps even central to his pluralistic philosophy of logic.[ii] Some years later Clarence Irwin Lewis furnished the axiomatics for several of these logics and since then the critics on the material implication have shown an increasing interest in these modal logics called “non-normal”. When Saul Kripke studied their semantics of “impossible worlds” as a way to distinguish between “necessity” and “validity” these logics reached a status of some respectability.[iii] As is well known, around the 70s non-normal logics were associated with the problem of omniscience in the epistemic interpretation of modal logic, specially in the work of Jaakko Hintikka and Veikko Rantala.[iv] Actually impossible worlds received a intensive study and development too in the context of relevant and paraconsistent logics - specially within the “Saint-Andrews- Australasian connection” in the work of such people as Graham Priest, Stephen Read, Greg Restall and Richard Routley-Sylvan. Nowadays, though the association with omniscience seems to have faded out, the study of non-normal logics has received a new impulse motivated through the study of counterlogicals.

The aim of the paper is to offer a dialogical interpretation of non-normal modal dialogics which will be connected to the discussion of counterlogicals as a minimalist defence of logical pluralism (pluralism for a monist) following the path prefigured by MacColl.

1. Would the real logic please stand up?

Conceiving situations in which not every mathematical or logical truth holds is a usual argumentation practice within formal sciences. However, to formulate the precise conditions which could render an adequate theory of logical arguments with counterpossibles in formal sciences is a challenging issue. Hartry Field has felt the need to tackle this challenge in the context of mathematics. Field writes:

> It is doubtless true that nothing sensible can be said about how things would be different if there were no number 17; that is largely because the antecedent of this counterfactual gives us no hints as to what alternative mathematics is to be regarded as true in the counterfactual situation in question. If one changes the example to 'nothing sensible can be said about how things would be different if the axiom of choice were false', it seems wrong …: if the axiom of choice were false, the cardinals wouldn't be linearly ordered, the Banach-Tarski theorem would fail and so forth [Field, 1989; pp.237]

These lines actually express the central motivation for a theory of counterpossibles in formal sciences. Namely, the construction of an alternative system where e.g. the inter-dependence of some axioms of a given formal system could be studied. If we were able to conceive not only a counterpossible situation where some axioms fail to be true but also even an alternative system without the axioms in question, then a lot of information could be won concerning the original "real" system. By the study of the logical properties of the alternative system we could e.g. learn which theorems of our "real system" are dependent on axioms missing in the alternative one.[V] Moreover, I would like to add that a brief survey of the history of mathematics would testify that this usage of counterpossibles seems to be a common practice in formal sciences.

The case of the study of counterpossibles in logic called *counterlogicals* is an exact analogue of the case of mathematics and motivates the study of alternative systems in the very same way. We learned a lot of intuitionistic logics, even the *insipiens* classical logical monist learned about his system while discussing with the antirealist. This seems to be a generally accepted fact, but why should we stop there? From free logics we learned about the ontological commitment of quantifiers, from paraconsistent logic ways of distinguishing between triviality and inconsistency;[vi] from connexive logics the possibility of expressing in the object language that a given atomic proposition is contingently true; from relevance logics that it is not always wise to distinguish between metalogical and logical "if, then"; from IF and epistemic dynamic logic we learned about arguments where various types of flow of information are at stake, for linear how to reason with limited resources, and so forth.

Are these alternative logics "real" or even the "true" logic? Well actually to motivate its study the mere mental construction of them is enough, the mere being *in intellectu*, provided such a construction is fruitful. I would even be prepared to defend t he that as a start it is enough if they teach us something about the logic we take to be the "real" one. The construction of alternative logics, which in the latter case is conceived as resulting from changes in the original "real" logic, can be thought of as following a substructural strategy: changes of logic are structural changes concerning logical consequence.

In the next chapter I will offer a dialogical interpretation of non-normal logics which should offer the first steps towards such a minimalist defence of logical pluralism. In this interpretation the pair *standard-non-standard* will be added to the pair "normal"-"non-normal". Furthermore, the adjectives *standard* and *non-standard* will qualify the noun *logic* rather than *world*, e.g. I will write "the standard logic Lk in the argumentative context m ". *Normal* will qualify those contexts, which do not allow the choice of a logic other than the standard one. Non-normal contexts do allow the choice of a new logic underlying the modalities of the chosen context. Before we go into the details let us distinguish between the following different kinds of counterlogical arguments:

1 Assume an intuitionist logician who puts forward the following conditional:
If tertium non-datur were valid in my logic, then the two sides of de Morgan Laws would hold (in my logic) too.

2 We take here once more our intuitionist
If tertium non-datur were valid in the non-standard logic Lk, then the two sides of de Morgan Laws would hold in Lk too.

In the first case the alternative logic -here classical logic- might be thought of as a conservative extension of the standard one here intuitionistic logic- i.e. any valid formula of the standard logic will be valid too in the non-standard logic. In the second case this seems to be less plausible: Lk could be a logic which is a combination of classical logic with some other properties very different from the intuitionistic ones. The situation is similar in the following cases where it is assumed that the standard logic is a classical one and the alternative logic can be a restriction:

3 *If tertium non-datur were not valid in my logic, then one side of de Morgan Laws would fail (in my logic).*

4 *If tertium non-datur were not valid in the non standard logic Lj, then one side of de Morgan Laws would fail (in Lj).*

Because of this fact it seems reasonable to implement the change of logics by means of a substructural strategy (akin to the concept of dialogics) - i.e. a strategy where the change of logics involves a change of the structural properties.[vii]

Now in these examples the precise delimitation of a logic is assumed as a local condition. However; the conditional involved in the counterlogical seems to follow another logic which would work as a kind of a metalogic that tracks the changes of the local assumption of a given logic while building arguments with such conditionals. The point here is that in this type of study classical logic has no privileged status. Classical logic might be "the metalogic" in many cases but certainly not here.

2 Non-normal dialogics

2.1 Motivation

Let us call *non-standard* such argumentation contexts (or "worlds") where a different logic holds relative to the logic defined as *standard*. Thus, in this interpretation of non-normal modal logic the fact that the law of necessitation does not hold is understood as implementing the idea that no logically valid argument could be proven in such systems to be unconditionally necessary (or true in any context and logic). Logicians have invented several logics capable of handling logically arguments that are aware of such a situation. The main idea of their strategy is simple: logical validity is about standard logics and not about the imagined construction of non-standard ones; we only have to restrict our arguments to the notion of validity involved in the standard logic. Actually there is a less conservative strategy: namely, one in which a formula is said to be valid if it is true in all contexts whether they are ruled by a standard or a non-standard logic. The result is notoriously pluralistic: no logical argument could be proven in such systems to be unconditionally necessary.

Anyway if we have a set of contexts, how are we to recognise those underlying a standard logic? The answer is clear in modal dialogics if we assume that the players can not only choose contexts but also the (non-modal) logic which is assumed to underlie the chosen context. In this interpretation the Proponent fixes the standards, i.e. determines which is the (non-modal) standard logic underlying the modalities of a given context. However under given circumstances the Opponent might choose a context where he assumes that a (non-modal) logic different from the standard one is at work. Now, there are some natural restrictions on the Opponent choices. Assume that in a given context **O** has *explicitly conceded* that **P** fixes the standards. In other words, the Opponent concedes that the corresponding formulae are assumed to hold under those structural conditions which define the standard logic chosen by the Proponent: we call these contexts *normal*. Thus, **O** has conceded that the context is normal - or rather, that the conditions in the context are normal. In this case **O** cannot choose the logic: it is **P** who decides which logic should be used to evaluate the formulae in question, and as already mentioned, **P** will always choose the logic he has fixed as the standard one. That is what the concession means: **P** has the choice.

Notice once more that "standard" logic does not really simply stand for "normal": normality, in the usual understanding of non-normal modal logic, is reconstructed here as a condition which when a context m is being chosen restricts the choice of the logic underlying the modalities of m.

2.2 Dialogics for S.05, S.2 and S3

The major issue here is to determine dynamically – i.e., during the process of a dialogue – in which of the contexts may the Opponent not have to conceded that it is a non-normal one and allowing him thus to choose a non-modal propositional logic different of the standard one. This must be a part of the dialogue's structural rules (unless we are not dealing with dialogues where the dialogical contexts with their respective underlying propositional logic are supposed to have been given and classified from the start). I will first discuss the informal implicit version of the corresponding structural rules and in the following chapter we will show how to build tableaux which implement these rules while formulating the notion of validity for the non-normal dialogics. Let us formulate a general rule implementing the required dynamics but some definitions first:

Or in the more formal notation of state of game (see appendix):

-particle rule: From A follows <R, σ, A, $\lambda^*A_{Lj\,/}\,n$>, responding to the attack $?_{/\,Lj}\,n$ stated by the challenger at m (underlying the logic ***Lk***) and where $\lambda^*A_{L/}n$ is the assignation of context n (with logic ***Lj***) to the formula A, and **n and *Lj*** are chosen by the challenger.

Definitions:

- *Normality as condition*: We will say that t a given context m is *normal* iff it does not allow to choose a (propositional) logic underlying the modalities of m other than the standard one. Dually a context is *non-normal* iff it does allow the choice of a new logic
- *Standard logic*: **P** fixes the standards, i.e. **P** fixes the (propositional) logic which should be considered as the standard logic underlying modalities and relative to which alternatives might be chosen.
- *Closing dialogues*:
- No dialogue can be closed with the moves (P)*a* and (O)*a* if these moves correspond to games with different logics
- *Particle rules for non-normal dialogics*:
  The players may choose not only contexts they may also choose the propositional logic underlying the modalities in the chosen contexts:

, ◊	*Attack*	*Defence*
***A* m** (Ahas been stated at context munderlying a logic L*k*)	? n_{Lj} m (at the context mthe challenger attacks by choosing an accessible context nand logic *Lj*)	A_{Lj} n
◊*A* m (◊A mhas been stated at context m underlying a logic *Lk*)	?◊ m	A_{Lj} n (the defender chooses the accessible context nand the logic *Lj*)

◊-*particle rule*: From ◊*A* follows < R, σ, *A*, λ**A* $_{Lj\ /}$ n>, responding to the attack ? ◊ n stated by the challenger at m (underlying the logic *Lk*) and where λ**A* $_{L/}$ n is the assignation of context n (with logic *Lj*) to the formula *A*, and n and *Lj* are chosen by the defender.

The accessibility relation is defined by appropriate structural rules fixing the global semantics (see appendix). To produce non-normal modal dialogic we proceed by adding the following (structural) rule:

(SR-ST10.O5) (SO5-rule):

- **O** may choose a non-standard logic underlying the modalities while choosing a (new) context n with an attack on a Proponent's formula of the form ˇ*A* or with a defence of a formula of the form ◊*A* stated in m if and only if m is non-normal.
- **P** chooses when the context is normal and he will always choose the standard logic but he may not change the logic of a given context (generated by the Opponent)
- The logic underlying the modalities of the initial context is assumed to be the standard logic.

O wins by choosing in 3 the structural rule, which changes the standard logic into an intuitionistic logic.

Let us produce a dialogical reconstruction of another logic, known as S2, where we assume not only that the logic of the first context is normal and in general SR-

Three further assumptions will complete this rule:

SO5 assumptions

(i) The dialogue's initial context has been assumed to be normal.
(ii) The standard logic chosen by **P** is classical logic ***Lc***.
(iii) No other context than the initial one will be considered as been normal.

The dialogic resulting from these rules - combined with the rules for T - is a dialogical reconstruction of a logic known in the literature as S.O5. In this logic validity is defined relative to the standard logic being classical and has the constraint that any newly introduced context could be used by **O** to change the standards. Certainly ˘$(a \quad \neg a)$ will be valid. Indeed, the newly generated context, which has been introduced by the challenger while attacking the thesis, has been generated from the normal starting context and thus will underlie the classical structural rule SR-ST2C (see appendix). The formula $(a \quad \neg a)$ on the contrary will not be valid. **P** will lose if **O** chooses in the second context, e.g., the intuitionistic structural rule SR-ST2I:

contexts	**O**		**P**			*contexts*
				$(a \vee \neg a)$	0	$1_{\{Lc\}}$
$1_{\{Lc\}}$	1	$<?_{/1.1}>$	0	$(a \vee \neg a)$	2	$1.1_{\{Lc\}}$
$1.1_{\{Li\}}$	3	$<?_{/1.1.1\{Li\}}>$	2	$a \vee \neg a$	4	$1.1.1_{\{Li\}}$
$1.1.1_{\{Li\}}$	5	$< ?\vee >$	4	$\neg a$	6	$1.1.1_{\{Li\}}$
$1.1.1_{\{Li\}}$		a	6	–		$1.1.1_{\{Li\}}$
				The Proponent loses playing with intuitionistic rules		

ST10.O5, but also:

(SR-ST10.2) (S2-rule):

- If **O** has stated in a context m a formula of the form ˘A (or if **P** has stated in **m** a formula of the form ◊A), then the context m *can* be assumed to be normal**.** Let us call (**O**) ˘A and (**P)** ◊A *norma - lity formulae.*
- **P** will not change the logic of a given context but he might induce **O** to withdraw a choice of a non-standard logic by forcing him to concede that the context at stake is a normal one.
- A normal context can only be generated from a(nother) normal context.

The first two points establish that a formula like ˘B could be stated by **P** under the condition that another formula, say, ˇ*A* holds. In this case **O** will be forced to concede that the context is normal and this normality will justify the proof of *B* within the standard logic. The third point of the rule should prevent that this process of justification from becoming trivial: formulae such as (**P**) ◊*A* *m*, or (**O**) ◊*A* **m** should not yield normality if *m* is no normal themselves: the normality of **m** should come from "outside" the scope of (**P**) … *m* and (**O**) ◊… **m.**

This is, for our purposes, a more appealing logic than S.05 because it makes of the status of the contexts at stake a question to be answered within the dynamics of the dialogue. One can even obtain certain iterations such as $((a \rightarrow b) \rightarrow (a \rightarrow b))$ which is not valid in S.05, but is in S2: the first context underlies the standard classical logic by the second S.05 assumption, the second context too because **O** will concede ˇ*a* there. Now, because the second context has been Ls-conceded by **O**, he cannot choose a logic different of the classical one, and **P** will thus win. Adding transitivity to S2 renders S3.

2.3 Dialogics for E.05, E2 and E3

The point of the logics presented in the chapter before was not to ignore the non-standard logics, but only to take into consideration the standard one while deciding about the validity of a given argument. We will motivate here a less conservative concept, namely, one in which a formula is said to be valid if it is true in all contexts whether they are ruled by a standard or a non-standard logic. These logics are known as E. In no E system will ˇ*A* be valid for any formula *A*.

Suppose one modifies S.05 in such a way that no context is assumed to be normal and thus every modality will induce a change of logic. This logic, called E.05, is unfortunately not of great interest: a formula will be valid in E iff it is valid in non-modal logics (think of $(a \rightarrow b) \rightarrow (a \rightarrow b)$, which in this logic cannot be proven to be valid). Modality seems not be of interest there, and this logic can be thought of as a kind of a modal lower limit.

Now the elimination of the assumption that the first context is normal in S2 - that is, take SR-ST10.O5 and SR-ST10.2 but drop the first and third S0.5 assumptions - yields an interesting dialogic for our purposes. $(a \rightarrow b) \rightarrow (a \rightarrow b)$ is valid there, signalising a more minimal structural condition for the validity of this formula than K (for it does not even assume, as K does, that validity concerns only contexts with the same kind of logic). Similarly one could produce D versions, etc. Indeed E2 seems to be the appropriate language where the logical pluralist might explore the way to formulate statements of logical validity which do not assume a universal scope.

In fact, up to this point; this interpretation only offers a way to explore the scope of the validity of some arguments when confronted with counterlogical situations, where no middle term is to be conceived between what is to be considered standard and what not. Moreover, that a central aim of this dialogic is to explore fruitful counterlogicals seems not to have been implemented yet. In the next chapter I would like to suggest some further possible distinctions in order to perform this implementation.

2.4 Beyond non-normality

Let us take once more the following example, where the standard logic is classical logic:

> *If tertium non-datur were not valid in my logic, then one sense of double negation would fail (in my logic).*

One possible formalisation consists of translating not-valid by "non-necessary". Now the problem with this example is; that, if **P** does not change the logic; he can win the (negative) conditional in, say, **S2** in a trivial way. Indeed, **O** will attack the conditional conceding the protasis, **P** will answer with the apodosis and after the mutual attacks on the negation **P** will win defending *tertium non-datur* in classical logic. But then the argument seems not to be terribly interesting. This follows from the fact that in the interpretation displayed above **P** may not change the standard logic once it has been fixed. In general this is sensible because validity should be defined relative to one standard and we cannot leave it just open to just any change. Moreover, though there is some irrelevance there this irrelevance concerns only the formula conceded at the object language: in our case double negation. But what is relevant and is used is the concession that the standard logic is the one where the classical structural rule applies. Finally why should **P** change the logic if he can easily win in the one he defined as standard?

However, in order to implement the dialogic of counterlogicals, one could leave some degree of freedom while changing the logical standard without too much complexity and inducing a more overall relevant approach: a given standard logic may change into a restriction of this logic. In other words, the standard logic may be changed to a weaker logic where any of its valid formulae are also valid in the stronger one **P** first defined as standard. True, the problem remains that it does not seem plausible that **P** will do it on principle: on principle he wishes to win, and if the proof is trivial all the better for him. There are two possibilities:

One is to build a dialogue under conditions determining from the start which contexts are played under the standard logic and which are the ones where the restriction of the standard logic hold (fix a model).

The other is to leave **O** to choose a conservative restriction of the logic **P** first defined as standard.

(SR-ST10.2) *:

- If **O** has stated in a context m a formula of the form *A* (or if **P** has stated in m a formula of the form ◊*A*), then the context m *can* be assumed to be normal. In these cases **O** might choose once a restriction of the standard logic and **P** must follow in his choices the restrictions on the standard logic produced by **O**.

- A normal context can only be generated from a(nother) normal context.

In our example **O** will choose intuitionistic logic and there **P** will need the concession of double negation if he wants to prove *tertium non-datur*. One way to see this point is that **O** actually tests if in the substructural rules defining the standard logic there are not some redundancies. Perhaps a sublogic might be enough.

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For the example of this chapter this seems enough but one could even allow such restrictions in the case of the initial context in S.05. Moreover one could even drop the second S.05 assumption and let **P** choose an arbitrary standard logic. Take for example the case

If transitivity were not holding in my logic, then a → a would fail too (in my logic).

Suppose the standard logic is S4. We should use a notation to differentiate the modality which defines the standard logic and which is normal from the modalities which are used within the corresponding non-normal logic. Let us use "Δ" (or " ") for necessity (or possibility) in the standard logic. Furthermore let us use Blackburn's hybrid language to "propositionalise" the properties of the accessibility relation. We could thus write

¬ ($\nu_i \rightarrow$—n)$_i$ *(transitivity) (in my S4 logic)* → ¬ (Δ *a* → ΔΔ*a*) *(in my S4 logic).*

If SR-ST10.2* applies then the Opponent will choose, say, the logic K and the Proponent will win. In these types of dialogue the Opponent functions more constructively than in the sole role of a destructive challenger. In fact, the Opponent is engaged in finding the minimal conditions to render the counterlogical conditional.

Actually there has already been some work done concerning the dialogic adequate for seeking the minimal structural conditions for modal logic. The dialogues have been called *structure seeking dialogues* (SSD) and have been formulated in Rahman/Keiff [2004]. In these dialogues the "constructive" role of the Opponent is put into work explicitly.[viii]

Here is another kind of example:

> *If the principle of non-contradiction were not valid in my logic, then one sense of double negation would fail (in my logic).*

One other way to formalise this would be to put the negation inside the scope of the necessity operator:

> *If it were necessary that the principle of non-contradiction does not hold, then it would be necessary that one sense of double negation will fail.*

If we assume here too that SR-ST10.2* applies then the Opponent will choose some sort of paraconsistent logic (such as Sette's P1). Certainly, the Opponent will lose, anyway but other choices would lead to a trivial winning strategy of the Proponent.

If, instead of using SR-ST10.2*, we leave the choice of the standard logic open, **P** might choose any logic as standard and then it would seem that almost anything goes. It is perhaps not the duty of the logician to prevent this but the application of SR-ST10.2* and the corresponding SSD can help there, leaving the Opponent to search for the "right" the structural conditions under which the formula should be tested.

The point may be put in a different way. In the dialogues of the preceding chapters the role of the Opponent is to test if the thesis assumes surreptitiously that its validity holds beyond the limits of the standard logic. In this role the Opponent may choose any arbitrary logic without any constraints. Let us now assume, that the Opponent, still in the role already mentioned, comes to the conclusion that the thesis of the Proponent holds as it is. The Opponent can then play a slightly different role and explore the possibilities of another strategy: he might try to check if the standard logic chosen is not too strong concerning the thesis at stake. The latter is the aim of the structure seeking dialogues.

As discussed in the appendix mentioned, the strategy dialogical games introduced above furnish the elements for building a tableau notion of validity where every branch of the tableau is a dialogue. Following the seminal idea at the foundation of dialogic, this notion is attained via the game-theoretical notion of *winning strategy*. X is said to have a winning strategy if there is a function, which, for any possible Y-move, gives the correct X-move to ensure the winning of the game. To produce in this way tableauy form the dialogues described above is pretty straightforward and I will leave the details to be worked out by the reader

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Appendix:

A.I A brief survey of dialogic:

The aim here is to introduce very briefly the conceptual kernel of dialogic in the context of the dialogical reconstruction of first-order propositional calculus, in its classical and intuitionist versions. [ix]

Let our language **L** be composed of the standard components of first-order logic (with four connectives , , →, ¬, and two quantifiers ,), with small letters (*a*, *b*, *c*,…) for prime formulæ, capital italic letters (*A*, *B*, *C*, …) for formulæ that might be complex, capital italic bold letters (***A***, ***B***, ***C***, …) for predicators, let our constants be noted τ_i, where i N, and our variables the usual (*x*, *y*, *z*, …). We will also need some special force symbols: $?_{\dots}$ and $!_{\dots}$, where the dots stand for indices, filled with some adequate information that will be specified by appropriate rules. An *expression* of **L** is either a term, a formula or a special force symbol. **P** and **O** are two other special symbols of **L**, standing for the players of the games. Every expression *e* of our language can be augmented with labels **P** or **O** (written **P**-*e* or **O**-*e*, called (*dialogically*) *signed expressions*), meaning in a game that the expression has been played by **P** or **O** (respectively). We use X and Y as variables for **P**, **O**, always assuming X≠Y. Other more specific labels will be introduced where needed.

An *argumentation form* or *particle rule* is an abstract description of the way a formula, according to its principal logical constant, can be criticised, and how to answer the criticisms. It is abstract in the sense that this description can be carried out without reference to a determined context. In dialogic we say that these rules state the *local semantics*, for they show how the game runs locally, in the sense that what is at stake is only the critic and the answer to a given formula with one logical constant rather than the whole (logical) context where this formula is embedded. Hence, the particle rules fix the dialogical semantics of the logical constants of **L** in the following way:

(Where *A* and *B* are formulæ, and *A*(*x*/t) is the result of the substitution of t for every occurrence of the variable *x* in *A*.)

			→
assertion:	X-*A* *B*	X- *A* *B*	X-A →*B*
attack	Y-$?_L$, or Y-$?_R$	Y- ?	Y-*A*
defence	(respectively) X-Aor X-*B*	X-*A*, or X-*B* X-*B*	X-*B*

			¬
assertion	X- x*A*	X- x*A*	X-¬*A*
	for any τ Y may choose,		
attack	Y- ? /τ	Y- ?	Y-*A*
	for any τ chosen by Y	for any τ X may choose,	
defence	X-A(x/τ)	X-A(x/τ)	— (i.e. no defence)

One more formal way to stress the locality of the semantics fixed by the particle rules is to see these rules as defining a state of a (structurally not yet determined) game. Namely:

Definition (*state of the game*): A *state of the game* is an ordered triple <ρ, σ, *A*> where:

— ρ stands for a role assignment either R, from players X, Y to only one element of the set{?(attack), !(defence)} determining which player happens to occupy the challenger and which the defender role, or R', inverting the role assignment R of both players (e.g. if R(X)=? and R(Y)=!, then R'(X)=! and R'(Y)=?). The players perform their assigned role as challengers (defenders) by stating an attack (or asserting a defence) fixed by the corresponding rule.

— σ stands for an assignment function, substituting as usual individuals by variables.

— *A* stands for a dialogically labelled subformula *A* with respect to which the game will proceed.

Particle rules are seen here as determining which state of the game S'follows from a given state S without yet laying down the (structural) rules which describe the passage from S to S'. What state follows of S=<R, σ, *F*>for the X-labelled formula *F*?

— *Negation particle rule*: If *F* is of the form ÿ*A* then S'=<R', σ, *A*>, i.e. Y will have the role of defending *A* and X the role of (counter)attacking *A*.

— *Conjunction particle rule*: If *F* is of the form *A* *B* then S'=<R, σ, *A*> or S'' =<R, σ, *B*>, according to the choice of challenger R(Y)=? between the attacks $?_L$ and $?_R$.

— *Disjunction particle rule*: If *F* is of the form *A⁄B* then S'=<R, σ, *A*> or S'=<R, σ, *B*>, according to the choice of defender R(X)=!, reacting to the attack ?⁄ of the challenger R(Y)=?.

— *Subjunction particle rule*: If *F* is of the form *A*→*B*, then S'=<R', σ, *A*> and the game might proceed to the state S''=<R'', σ, *B*>, or even the other way round according to the choice of the defender and reacting to the attack *A* of the challenger R(X)=?.

— *Universal quantifier particle rule*: If *F* is of the form " *Ax* then S'=<R, σ(*x*/t), *A*> for any constant τ chosen by the challenger R(Y)=? while stating the attack ? /τ.

— *Existential quantifier particle rule*: If *F* is of the form *xAx* then S'=<R, σ(*x*/t), *A*> for any constant τ chosen by the defender R(X)=! reacting to the attack ? of the challenger R(Y)=?.

A dialogue can be seen as a sequence of labelled expressions, the labels carrying information on the game significance of these expressions. Dialogues are processes, so they are dynamically defined by the evolution of a game, which binds together all the labels mentioned. In other words, the set of expressions which is a complete dialogue can be dynamically determined by the rules of a game, specifying how the set can be extended from the original thesis formula. Particle rules are part of the definition of such a game, but we need to set the general organisation of the game, and this is the task of the *structural rules*. Actually structural rules can, while implementing the local semantics of the logical particles, determine a kind of game for a context where e.g. the aim is persuasion rather than logical validity. In these cases dialogic extends to a study of argumentation in a broader sense than the logical one. But *when the issue at stake is indeed testing validity*, i.e. when **P** can succeed with the use of the appropriate rules in defending the thesis against all possible allowed criticism by **O**, games should be thought of as furnishing the branches of a tree which displays the games relevant for testing the validity of the thesis. As a consequence of this definition of validity, each split of such a tree into two branches (dialogue games) should be considered as the outcome of a propositional choice of **O**. In other wordswhen **O** defends a disjunction, he reacts to the attack against a conditional, and when he attacks a conjunction, he chooses to generate a new branch (dialogue). Dually **P** will not choose to change the dialogue (branch). In fact, from the point of view of games as actual (subjective) procedures (acts), it could happen that the subject playing as **O** (**P**) is not clever enough to see that his best strategy is to open (not to open) a new dialogue game (branch) anytime he can, but in this context where the issue is an inter-subjective concept of validity, which should lead to a straightforward construction of a system of tableaux, we simply assume that **O** makes the best possible move.

(SR-ST0) (*starting rule*): Expressions are numbered and alternately uttered by **P** and **O**. The thesis is uttered by **P.** All even-numbered expressions including the thesis are **P**-labelled, all odd-numbered expressions are **O** moves. Every move below the thesis is a reaction to an earlier move with another player label and performed according to the particle and the other structural rules.

(SR-ST1) (*winning rule*): A dialogue is closed iff it contains two copies of the same prime formula, one stated by **X** and the other one by **Y**, and neither of these copies occur within the brackets "<" and ">" (where any expression which has been bracketed between these signs in a dialogue either cannot be counterattacked in this dialogue, or it has been chosen in this dialogue not to be counterattacked). Otherwise it is open. The player who stated the thesis wins the dialogue iff the dialogue is closed. A dialogue is finished if it is closed or if no other move is allowed by the (other) structural and particle rules of the game. The player who started the dialogue as a challenger wins if the dialogue is finished and open.

(SR-ST2I) (*intuitionist ROUND closing rule*): In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against *the last not already defended* attack. Defences may be postponed as long as attacks can be performed. Only the latest open attack may be answered: if it is **X**'s turn at position n and there are two open attacks m, l such that $m < l < n$, then **X** may not at position n defend himself against m.

(SR-ST2C) (*classical ROUND closing rule*): In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against *any attack* (including those which have already been defended).

(SR-ST3/SY) (*strategy branching rule*): At every propositional choice (i.e., when X defends a disjunction, reacts to the attack against a conditional or attacks a conjunction), X may motivate the generation of two dialogues differentiated only by the expressions produced by this choice. X might move into a second dialogue iff he loses the first chosen one. No other move will generate new dialogues.

(SR-ST4) (*formal use of prime formulæ*): **P** cannot introduce prime formulæ: any prime formula must be stated by **O** first. Prime formulæ can not be attacked.

(SR-ST5) ***(no delaying tactics rule)***:

While playing with the classical structural rule **P** may perform once a new defence (attack) of an existential (universal) quantifier using a different constant (but not new) iff the first defence (attack) compelled **P** to introduce a new constant. No other repetitions are allowed.

While playing with the intuitionistic structural rule **P** may perform a repetition of an attack if and only if **O** has introduced a new prime formula which can now be used by **P.**

Definition (***Validity***): A tableau for (**P**)A (i.e. starting with (**P**)A) proves the validity of A iff the corresponding tableau is closed. That is, iff every dialogue generated by (**P**)A) is closed.

Examples: In Fig. 1 the outer columns indicate the numerical label of the move, the inner columns state the number of a move targeted by an attack. Expressions are not listed following the order of the moves, but writing the defence on the same line as the corresponding attack, thus showing when a round is closed. Recall, from the particle rules, that the sign "—" signalises that there is no defence against the attack on a negation. In this example **P** wins because, after the **O**'s last attack in move 3, **P**, according to the (classical) rule ST2C, is allowed to defend himself (once more) from the attack in move 1 (in the same dialogue). **P** states his defence in move 4 though, actually, **O** did not repeat his attack – this fact has been signalised by inscribing the unrepeated attack between square brackets.

O			**P**		
				$a \vee \neg a$	0
1	?	0		$\neg a$	2
3	a	2		—	
[1]	[?]	[1]		a	4

Fig. 1. SDC rules. **P** wins.

O			**P**		
				$a \vee \neg a$	0
1	?	0		$\neg a$	2
3	a	2		—	

Fig. 2. SDI rules. **O** wins.

In the game of Fig. 2, **O** wins because, after the challenger's last attack in move 3, **P**, according to the intuitionistic rule SR-I, is not allowed to defend himself (once more) from the attack in move 1.

Philosophical remarks: games as propositions.

Particle rules determine dynamically how to extend a set of expressions from an initial assertion. In the game perspective, one of the more important features of these rules is that they determine, whenever there is a choice to be made, who will choose. This is what can be called the pragmatic dimension of the dialogical semantics for the logical constants. Indeed, the particle rules can be seen as a proto-semantics, i.e. a game scheme for a not yet determined game which when completed with the appropriate structural rules will render the game semantics, which in turn will build the notion of validity.

Actually by means of the particle rules games have been assigned to sentences (that is, to formulæ). But sentences are not games, so what is the nature of that assignment? The games associated to sentences are meant to be *propositions* (i.e. the constructions grasped by the (logical) language speakers). What is connected by logical connectives are not sentences but propositions. Moreover, in the dialogic, logical operators do not form sentences from simpler sentences, but games from simpler games. To explain a complex game, given the explanation of the simpler games (out) of which it is formed, is to add a rule which tells how to form new games from games already known: if we have the games *A* and *B*, the conjunction rule shows how we can form the game *A*Ÿ*B* in order to assert this conjunction.

Now, particle rules have another important function: they not only set the basis of the semantics, and signalise how it could be related to the world of games – which is an outdoor world if the games are assigned to prime formulæ, but they also show how to perform the relation between sentences and propositions. Sentences are related to propositions by means of assertions, the content of which are propositions. Assertions are propositions endowed with a theory of force, which places logic in the realm of linguistic actions. The forces performing this connection between sentences and propositions are precisely the attack (?) and the defence (!). An attack is a demand for an assertion to be uttered. A defence is a response (to an attack) by acting so that you may utter the assertion (e.g. that *A*). Actually the assertion force is also assumed: utter the assertion that *A* only if you know how to win the game *A*.

Certainly the "know" introduces an epistemic moment, typical of assertions made by means of judgements. But it does not presuppose in principle the quality of knowledge required. The constructivist moment is only required if the epistemic notion is connected to a tight conception of what means that the player X *knows that there exists* a winning game or strategy for *A.*

NOTES

i Unfortunately he does not seem to have succeeded. Read [1998], differs from Storrs MacCall's ([1963] and [1967]) argues that the reconstruction of MacColl's modal logic yields T and not one of the non-normal logics.

ii Cf. Grattan-Guinness [1998], Rahman [1997], [1997], [1998] [2000], Read [1998] and Wolenski [1998].

iii Cf. Kripke [1965].

iv Cf. Hintikka [1975] and Rantala [1975]. See too Cresswell [1972] and Girle [1973].

v See too Read [1994], 90-91 and Priest [1998], 482.

vi Already Aristoteles used counterlogical arguments while studying the principle of non-contradiction, which he saw as the principal axiom of logic.

vii This strategy, as developed in Rahman/Keiff [2003], could be implemented either implicitly or explicitly. The implicit formulation presupposes that the structural rules are expressed at a different level than the level of the rules for the logical constants which are part of the object language. The explicit formulation renders a propositionalisation of the structural rules using either the language of the linear logicians or hybrid languages in the way of Blackburn [2001].

viii In the context of the SSD with the thesis; say, *A*, the Proponent's claims that he assumes that a determined element d_i (of a given set D of structural rules) is the minimal structural condition for the validity of *A*. Informally, the idea is that structural statements can be attacked by the challenger in two distinct ways. *First*, by conceding the condition d_i, claimed by the player X to be minimal, and asking X to prove the thesis. *Second*, by (counter)claiming that the thesis could be won with a (subset of) condition(s) of lesser rank in D. In that case, the game proceeds in a subdialogue, started by the challenger who now will claim that the formula in question can be won under the hypothesis d_j, where d_j is different from d_i and has a lesser rank as d_i. Since the challenger (Y) starts the subdialogue *he now has to play formally*. See details in Rahman/Keiff [2003].

ix Cf. Lorenzen [1958] and Lorenzen/Lorenz [1978]. The present more modern version stems from Rahman/Keiff [2004].

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